University of California, Berkeley
Physics 110A Fall 2001 Section 1 (*Strovink*)

FINAL EXAMINATION

Directions: Do all six problems, which have unequal weight. This is a closed-book closed-note exam except for three $8\frac{1}{2} \times 11$ inch sheets containing any information you wish on both sides. A photocopy of the four inside covers of Griffiths is included with the exam. Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (30 points)
A conducting sphere of radius a, centered at the origin, carries a constant total charge q_0. Outside it, between radii $r = a$ and $r = b$, lies a spherical shell composed of an insulating dielectric with "frozen-in" polarization

$$\mathbf{P} = \hat{r}\frac{q_0}{4\pi r^2} ,$$

where q_0 is the same constant.

Calculate the electric field $\mathbf{E}(r)$ over the entire region $0 < r < \infty$. (Note that the dielectric constant ϵ is not defined or supplied here, and should not appear in your answer.)

Problem 2. (30 points). **Static fields**.
Please write down the following simple static electromagnetic fields in vacuum. Credit will be based entirely upon your answer; to receive any credit, your answer must be exactly correct, including the field's direction as well as its magnitude.

(**a**) (5 points)
$\mathbf{E}$ inside a parallel plate capacitor that holds charges $\pm q$ on plates of area a located at $y = \pm\frac{d}{2}$. (Note that the positive plate is on top.)

(**b**) (5 points)
$\mathbf{E}$ a distance r from the origin, where r is outside a spherically symmetric distribution of total charge Q that is centered at the origin.

(**c**) (5 points)
$\mathbf{E}$ at $(x \neq 0, 0, 0)$ produced by an ideal electric dipole of moment $\mathbf{p} = p_0\hat{z}$ that is centered at the origin.

(**d**) (5 points)
$\mathbf{B}$ a distance s outside a long thin wire carrying current I along $\hat{z}$.

(**e**) (5 points)
$\mathbf{B}$ at the center $(0, 0, 0)$ of a circular wire loop of radius b lying in the xy plane, carrying counterclockwise current I.

(**f**) (5 points)
$\mathbf{A}$ inside a long circular cylinder with its axis along $\hat{z}$, containing a magnetic field $\mathbf{B} = B_0\hat{z}$ inside, and $\mathbf{B} = 0$ outside.

Problem 3. (30 points)
A long thick cylindrical wire of radius b carries a steady current I_0 along its axis $\hat{z}$, uniformly distributed over the wire's cross section. At $t = 0$ the wire is cut with a thin saw to produce a thin gap in the region $-\frac{d}{2} < z < \frac{d}{2}$, with $d \ll b$. Neglect fringing effects near $s = b$.

(**a**) (15 points)
For a period of time after $t = 0$, the power supply that is connected to the distant ends of the wire forces the same current I_0 to continue to flow in the wire. Calculate the magnitude and direction of the magnetic field in the gap.

(**b**) (15 points)
At some later time, a resistor is substituted for the power supply, and the charge that accumulated on the faces $z = \pm\frac{d}{2}$ is allowed to drain away. While this charge is draining, would you expect the electric field in the gap to continue to be exactly uniform (independent of s)? Why or why not?

Problem 4. (40 points)
A nonrelativistic electron of mass m and charge e moves in vacuum in the xy plane under the influence of a constant uniform magnetic field B that is directed along the z axis. Because no other externally applied fields or mechanical forces exist, the electron travels very nearly in a periodic orbit. After one revolution, it is observed that the electron's kinetic energy has diminished slightly, by a factor $1-\eta$, where $\eta \ll 1$.

In terms of B and fundamental constants, calculate η.

Problem 5. (30 points)
A plane wave of wavelength λ is normally incident on a system of thin slits at constant y in the aperture plane $z=0$. An observer at $z=\infty$ observes the Fraunhofer-diffracted beam at a small angle $\theta \equiv \arctan(dy/dz)$. In each part of this problem, you are asked to calculate the diffraction-pattern ratio

$$R(\theta) \equiv \frac{I(\theta)}{I(0)} ,$$

where the intensity I is proportional to the square of the wave amplitude, *i.e.* to the time-averaged Poynting vector.
(**a**) (10 points)
Write down $R(\theta)$ for two thin slits at $y = \pm a/2$.
(**b**) (10 points)
Write down $R(\theta)$ for four thin slits, two at

$$y = +(a \pm b)/2 ,$$

and two at

$$y = -(a \pm b)/2 ,$$

where $a > b > 0$.
(**c**) (10 points)
Take the incident beam to be circularly polarized. Repeat part (**b**) under the same conditions, except that an $\hat{x}$ polarizer is placed behind the top pair of slits, and an otherwise identical $\hat{y}$ polarizer is placed behind the bottom pair.

Problem 6. (40 points)
A waveguide consists of an evacuated rectangular pipe that runs parallel to the $\hat{z}$ axis. The pipe has three perfectly conducting metal sides, at $x=0$, $x=a$, and $y=0$. These three sides are connected together in a "U" shape. The fourth (top) side, at $y=b$, is made of the same material but is insulated from the "U".

Operating in the TEM mode, the waveguide carries an electromagnetic pulse that travels in the $+\hat{z}$ direction. At $z=0$ and $t=0$, a snapshot is taken of the (nonzero) magnetic field $\mathbf{B}(x,y,z=0,t=0)$. Calculate $\mathbf{B}(x,y,z=0,t=0)$ within a multiplicative constant.